GOSFORD HIGH SCHOOL

MATHEMATICS

HSC

2009/2010

Assessment Task 1

Time Allowed: 70 minutes

<u>**Instructions:**</u>

- All questions are **NOT** of equal value
- Start each **QUESTION** on a new page
- Write on one side of the page only
- Write your name on every page
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work
- Board approved calculators may be used in all sections

Question 1: (12 Marks) Start a New Page for this Question

a) Find $\frac{d}{dx}(1+6x^2)^2$ — 1 Mark

b) Find $\frac{dy}{dx}$ if $y = \frac{3x+4}{1-x^3}$ — 2 Marks

c) Differentiate $x^3(3-2x)^3$. Write answer in simplest factored form — 3 Marks

d) Differentiate $2x^3 - \frac{5}{x} - 3 - 2\sqrt{x}$ — 2 Marks

e) i) Calculate: — 2 Marks

$$\lim_{x \to -2} \frac{6x^2 + 21x + 18}{x+2}$$

ii) A curve $y = f(x)$ is defined as — 2 Marks

$$f(x) = \begin{cases} \dfrac{6x^2 + 21x + 18}{x+2} & for\ x < -2 \\ x^2 - 1\ for\ x \geq -2 \end{cases}$$

Is the curve continuous at $= -2$? Give reasons for your answer.

Question 2: (11 Marks) Start a New Page for this Question

a) Find the centre and radius of a circle with equation — 3 Marks

$$x^2 - 2x + y^2 + 6y + 9 = 0$$

b) A and B are the points $(-2,0)$ and $(0,-2)$ respectively. — 3 Marks
Find the equation of the locus of the point $P\ (x,y)$ which moves so that the distance from A to P is three times the distance from B to the point P.

c) For the points $A\ (2,-1)$ and $B\ (0,1)$ show that the equation of the locus of a point P which moves so that PA and PB are at right angles is $x^2 - 2x + y^2 = 1$ — 2 Marks

d) Find the equation of the locus of a point $P\ (x,y)$ which moves so that it is equidistant from the point $(-2,2)$ and the line $x = 2$ — 3 Marks

Question 3: (12 Marks) Start a New Page for this Question

a) For the parabola $y = 4x^2$, find:

i) the equation of the directrix — 2 Marks

ii) the coordinates of the focus — 1 Mark

b) For the parabola $y = -\frac{1}{2}(x+2)^2 + 1$

i) Find the coordinates of the vertex — 2 Marks

ii) Find the coordinates of the focus — 2 Marks

c) Write down the equation of the parabola with focus at (2,4) and directrix $y = -2$. — 2 Marks

d) Find the focal length of the parabola with axis of symmetry $x = 2$ and vertex $(2, -3)$ which passes through the point $(4, -6)$ — 3 Marks

Question 4: (15 Marks) Start a New Page for this Question

a) Consider the curve

$$y = x^3 - 9x^2 + 15x + 10$$

i) Find the coordinates of the stationary points on the curve. — 3 Marks

ii) Determine the nature of each of these stationary points. — 2 Marks

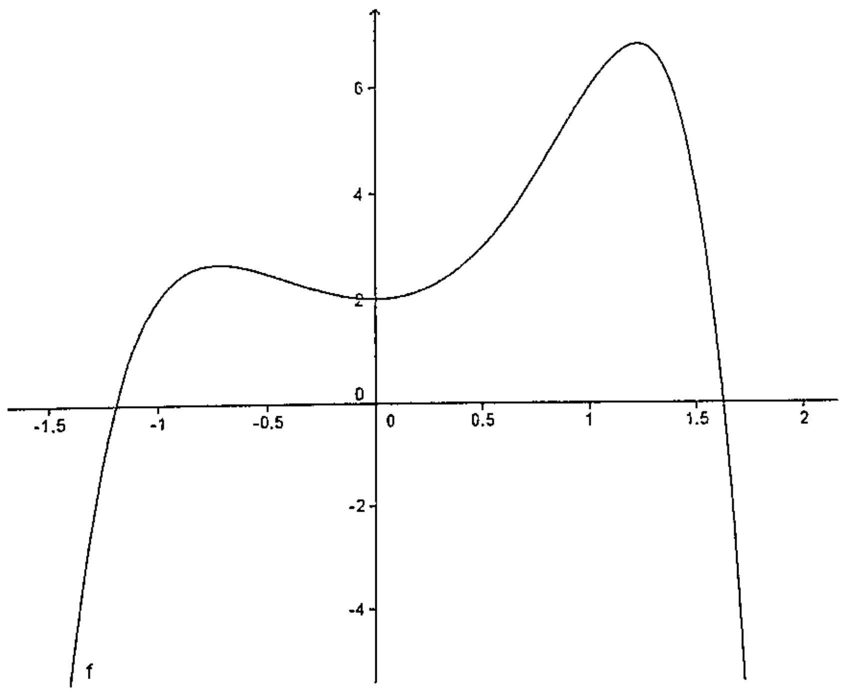
iii) Find the coordinates of the point of inflexion — 2 Marks

iv) Neatly sketch the curve for $0 \leq x \leq 7$, clearly labelling all important features including the y-intercept. — 2 Marks

v) What are the greatest and least values of the function in the interval $0 \leq x \leq 7$ — 1 Mark

b) Given $f(x) = x^3 - 6x^2 + 9x - 5$, find for what values of x the function is increasing — 2 Marks

c) Below is a graph of $y = f(x)$. Trace or copy the diagram to your answer sheet and on the same number plane neatly sketch $y = f'(x)$ showing all important features 3 Marks



Question 5: (11 Marks) Start a New Page for this Question

a) At what x-values on the curve $y = x^3 - 6x^2 - 30x + 5$ is the tangent to the curve parallel to the line $y = 6x - 5$ 2 Marks

b) A continuous curve $y = f(x)$ has the following properties for the interval $a \leq x \leq b$: 3 Marks

$$f(x) > 0, f'(x) < 0 \text{ and } f''(x) > 0$$

Sketch a curve satisfying these conditions and state the greatest value of $f(x)$ in this interval.

c) A quantity (Q) is decreasing at a decreasing rate. 2 Marks

Describe the signs of $\frac{dQ}{dt}$ and $\frac{d^2Q}{dt^2}$ where t is time

d) i) Show that the curve $f(x) = \sqrt{1-2x}$ has no stationary points 2 Marks

ii) Determine the concavity of $f(x)$ for all x values 2 Marks

Assessment Task 1

1) a) $2(1+6x^2)' \times 12x = 24x(1+6x^2)$

b) $y' = \frac{(1-x^3)\times 3 - (3x+4)\times -3x^2}{(1-x^3)^2}$

$= \frac{3-3x^3+9x^3+12x^2}{(1-x^3)^2}$

$= \frac{6x^3+12x^2+3}{(1-x^2)^2}$

c) $x^3 \times 3(3-2x)^2 \times -2 + (3-2x)^3 \times 3x^2$

$= (3-2x)^2(-6x^3+9x^2-6x^3)$

$= 3x^2(3-2x)^2(3-4x)$

d) $y = 2x^3 - 5x^{-1} - 3 - 2x^{1/2}$

$y' = 6x^2 + 5x^{-2} - x^{-1/2}$

$= 6x^2 + \frac{5}{x^2} - \frac{1}{\sqrt{x}}$

e) i) $\lim_{x\to -2} \frac{3(2x^2+7x+6)}{x+2}$

$= \lim_{x\to -2} \frac{3(2x+3)(x+2)}{x+2}$

$= 3(2\times -2+3)$

$= -3$

ii) $f(-2) = (-2)^2 - 1$

$= 3$

Since $f(-2) \neq \lim_{x\to -2}$ the function is not continuous

2) a) $x^2-2x+1+y^2+6y+9 = 0+$

$(x-1)^2+(y+3)^2 = 1$

centre: $(1, -3)$

radius: 1

b) $PA = \sqrt{(x+2)^2+y^2}$

$PB = \sqrt{x^2+(y+2)^2}$

$3PB = PA$

$9(x^2+(y+2)^2) = (x+2)^2+y^2$

$9x^2+9y^2+36y+36 = x^2+4x+4+y^2$

$8x^2-4x+8y^2+36y = -32$

c) $m_{PA} = \frac{y+1}{x-2}$ $m_{PB} = \frac{y-1}{x}$

For a right angle

$m_1 m_2 = -1$

$\frac{y+1}{x-2} \times \frac{y-1}{x} = -1$

$y^2 - 1 = -x^2+2x$

$x^2-2x+y^2 = 1$

d) focus: $(-2, 2)$

focal length $= 2$

vertex: $(0, 2)$

$(y-2)^2 = 4a(x-0)$

$(y-2)^2 = -8x$

3) a) i) focal length $= \frac{1}{16}$

$y = -\frac{1}{16}$

ii) focus $= (0, \frac{1}{16})$

b) i) $(-2, 1)$

ii) $2(y-1) = (x+2)^2$

$a = \frac{1}{2}$

focus $= (-2, \frac{1}{2})$

c) focal length $= 3$

$(x-2)^2 = 12(y-1)$

d) $(x-2)^2 = 4a(y+3)$

$(4-2)^2 = 4a(-6+3)$

$4 = -12a$

$a = -\frac{1}{3}$

$(x-2)^2 = -\frac{4}{3}(y+3)$

4) a) i) $y = x^3-9x^2+15x+10$

$y' = 3x^2-18x+15$

$y'' = 6x-18$

$3(x^2-6x+5) = 0$

$(x-5)(x-1) = 0$

$x=1$ $x=5$

$y=17$ $y=-15$

ii) $(1,17)$ is a max

$(5,-15)$ is a min

iii) $6x-18=0$

$x=3$

$y=1$

test y'' on both sides

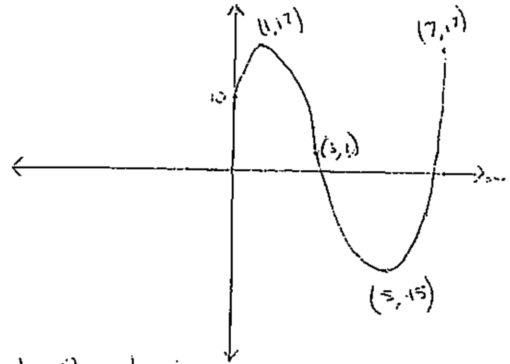
x	2	3	4
y''	-6	0	6

$\therefore (3, 1)$ is a point of inflexion

iv) at $x=0$ $y=10$

$x=7$ $y=17$

y-intercept is 10



v) Greatest 17

Least -15

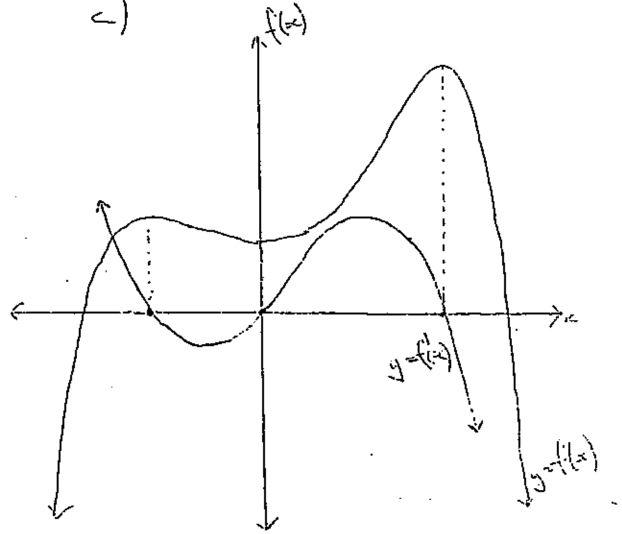
b) $f'(x) = 3x^2-12x+9$

$3(x^2-4x+3) > 0$

$3(x-3)(x-1) > 0$

$x<1$ or $x>3$

c)



5) a) $y' = 3x^2 - 12x - 30$

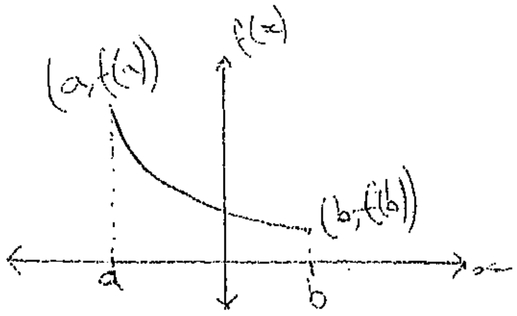
$3x^2 - 12x - 30 = 6$

$3(x^2 - 4x - 12) = 0$

$(x-6)(x+2) = 0$

$x = -2$ or $x = 6$

b)



Greatest value $= f(a)$

c) $\frac{dQ}{dt} < 0 \quad \frac{d^2Q}{dt^2} > 0$

d) i) $f'(x) = \frac{1}{2}(1-2x)^{-\frac{1}{2}} \times -2$

$\frac{-1}{\sqrt{1-2x}} = 0$

$-1 = 0$

$\therefore$ no stationary points

ii) $f''(x) = \frac{1}{2}(1-2x)^{-\frac{3}{2}} \times -2$

$= -\frac{1}{(\sqrt{1-2x})^3}$

since $\sqrt{1-2x}$ is always positive

$f''(x)$ is always negative

So $f(x)$ is concave down

Question 5

a) $y = x^3 - 6x^2 - 30x + 5$

Where is $y' = 6$

$y' = 3x^2 - 12x - 30 = 6$

$3x^2 - 12x - 36 = 0$

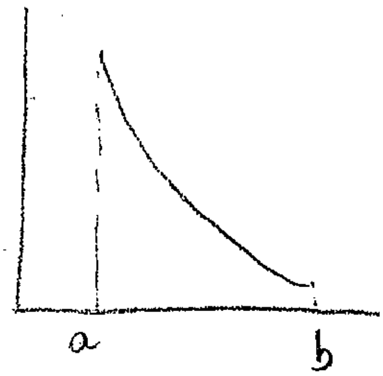
$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6$ or -2

(2)

b)



Greatest value is $f(a)$

least value is $f(b)$

(3)

c) $\frac{dQ}{dt} < 0$ $\quad$ $\frac{d^2Q}{dt^2} < 0$

(2)

, OR

d) $f(x) = (1-2x)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(1-2x)^{-\frac{1}{2}} \cdot -2 = \frac{-1}{\sqrt{1-2x}}$ (

For stationary pts, $y' = 0$

$\frac{-1}{\sqrt{1-2x}} = 0$ No solutions

ii) $f''(x) = -1 \times -\frac{1}{2}(1-2x)^{-\frac{3}{2}} \cdot -2 = \frac{-1}{\sqrt{(1-2x)^3}}$

$f''(x)$ is always negative in its domain

Domain $x \leq \frac{1}{2}$. But $f''(x)$ undefined at $x = \frac{1}{2}$

$\therefore$ Curve concave down for $x < \frac{1}{2}$ (i

$\therefore$ infinite gradient at $x = \frac{1}{2}$. No concavity

Curve does not exist for $x > \frac{1}{2}$